

# Cambridge International AS & A Level

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## MATHEMATICS

9709/32

### Paper 3 Pure Mathematics 3

October/November 2023

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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- 1 (a) Sketch the graph of  $y = |4x - 2|$ .

[1]

- (b) Solve the inequality  $1 + 3x < |4x - 2|$ .

[4]

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2 The parametric equations of a curve are

$$x = (\ln t)^2, \quad y = e^{2-t^2},$$

for  $t > 0$ .

Find the gradient of the curve at the point where  $t = e$ , simplifying your answer.

[4]

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- 3 The polynomial  $2x^3 + ax^2 - 11x + b$  is denoted by  $p(x)$ . It is given that  $p(x)$  is divisible by  $(2x - 1)$  and that when  $p(x)$  is divided by  $(x + 1)$  the remainder is 12.

Find the values of  $a$  and  $b$ .

[5]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 4 (a) On a sketch of an Argand diagram, shade the region whose points represent complex numbers  $z$  satisfying the inequalities  $|z - 4 - 3i| \leq 2$  and  $\operatorname{Re} z \leq 3$ . [4]

- (b) Find the greatest value of  $\arg z$  for points in this region. [2]

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5 Find the exact value of  $\int_0^6 \frac{x(x+1)}{x^2+4} dx$ .

[6]

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- 6 (a)** By sketching a suitable pair of graphs, show that the equation

$$\cot x = 2 - \cos x$$

has one root in the interval  $0 < x \leq \frac{1}{2}\pi$ .

[2]

- (b)** Show by calculation that this root lies between 0.6 and 0.8.

[2]

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- 7 (a)** By expressing  $3\theta$  as  $2\theta + \theta$ , prove the identity  $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$ . [3]

[illegible]

**(b)** Hence solve the equation

$$\cos 3\theta + \cos \theta \cos 2\theta = \cos^2 \theta$$

for  $0^\circ \leq \theta \leq 180^\circ$ .

[5]

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**8** It is given that  $\frac{2+3ai}{a+2i} = \lambda(2-i)$ , where  $a$  and  $\lambda$  are real constants.

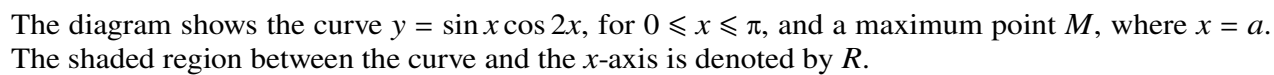
(a) Show that  $3a^2 + 4a - 4 = 0$ .

[4]

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- (b)** Hence find the possible values of  $a$  and the corresponding values of  $\lambda$ . [3]

[illegible]



- [illegible]

**[Turn over**

where  $c$  is a positive constant. It is given that the angle between  $l$  and  $m$  is  $60^\circ$ .

- [4]

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**[Turn over**

- 11 The variables  $x$  and  $y$  satisfy the differential equation

$$x^2 \frac{dy}{dx} + y^2 + y = 0.$$

It is given that  $x = 1$  when  $y = 1$ .

- (a) Solve the differential equation to obtain an expression for  $y$  in terms of  $x$ . [8]

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- (b) State what happens to the value of  $y$  when  $x$  tends to infinity. Give your answer in an exact form. [1]

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[illegible]

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